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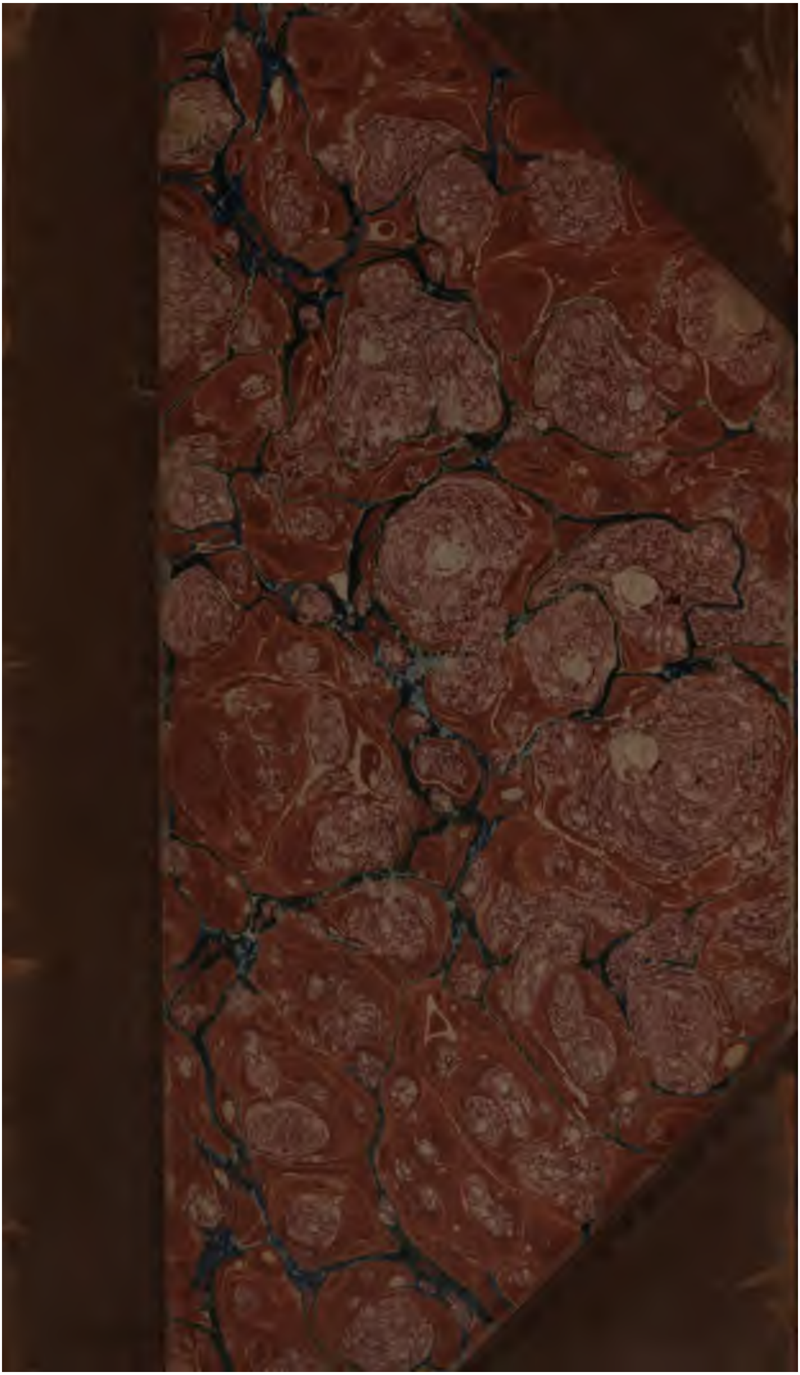

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ON

MUSICAL INTERVALS,

HARMONICS,

&c.

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ESSAY

ON

MUSICAL INTERVALS, HARMONICS,

AND THE

TEMPERAMENT OF THE MUSICAL SCALE,

&c.

BY W. S. B. WOOLHOUSE,

HEAD ASSISTANT ON THE NAUTICAL ALMANAC ESTABLISHMENT.

22

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PREFACE.

THE substance of the following Essay was written some years ago at the express wish of several musical friends; but its publication has been delayed by fortuitous circumstances, to which, however, it will not be necessary to advert.

There can be but little difference of opinion as to the practical, as well as theoretical, advantages to be derived from a correct knowledge of the natural relations that exist amongst the several sounds in the musical scale; and, to all who can fully appreciate these advantages, it must be an acquisition to possess a concise view of the most important principles and results. The Author is not aware of any work of this kind fully adapted to the increasing wants of such musicians as are actuated by a love of philosophical truth, a devotion to the science, and a desire to comprehend an exact knowledge of its resources.

In the present attempt to supply this deficiency, brevity has had the principal share of consideration, while the omission of any thing useful has been carefully avoided; and, though the work is compressed into very narrow limits, it is hoped that the practical musician will find in it the means of resolving any difficulty that may not have distinctly met with attention.

The calculation of the various combinations of musical intervals is much simplified when they can be arithmetically estimated in whole numbers. This object is attained without any sensible inaccuracy by dividing the octave into 730 equal intervals; and the values of the elemental intervals on this scale are given in article 14. As exercises on the use of these numbers in calculation, innumerable examples may occur, such as—What is the difference between twelve perfect fifths and seven octaves? What is the difference between three perfect major-thirds and an octave? What difference is there between two perfect major-thirds and a perfect minor-sixth? &c. All such questions are immediately answered by the most simple arithmetical calculation, and to a degree of accuracy more than sufficient to satisfy every reasonable scruple.

Most writings that have appeared on the theory of musical sounds are grounded on the principle of the combination of ratios. This principle, though conveniently applicable to the calculation of the divisions of a monochord which answer to the perfect intervals, presents but little facility in the detection of small intervals. On the other hand, the division of the octave into a number of equal parts affords a precise view of the relative values of all intervals whatever; the extent of each interval, which on the monochord is represented to the eye, is by this means comprehended mentally; and the ear of a singer or violin player becomes considerably assisted in the acquirement of an accurate execution of either true or tempered melody.

The theories of the temperaments of the musical scale, and of the interesting phenomenon of the beats of imperfect concords, are of much importance to tuners, and the most useful elements of these theories are laid down with every attention to simplicity and precision.

It will be quite unnecessary to state any details of the plan and pretensions of so small a work as the present, the object of which is merely to furnish the means of a first step towards the general inculcation of the scientific principles of music amongst amateurs.

London:
March 20, 1835.

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MUSICAL INTERVALS, &c.

SOUND.

1. Sound is an affection of the mind, produced by the action of rapid vibratory motions on the organs of hearing. All mediums are, in some degree, capable of propagating these vibrations, though the ambient air, or atmosphere, is naturally that by which they are usually transmitted to the ear. When the air is disturbed by the vibrations of a sonorous body, such as a string of a violin, or the parchment of a drum, waves are formed that proceed from the place of disturbance, as a centre, in all directions. There is no transmission of the particles of air, as in the case of wind; sonorous waves are sustained and propagated by a process of alternate compression and rarefaction amongst the progressive particles of the fluid. The moveable particles proceed in the direction of the transmission when under the influence of compression, and return as soon as rarefaction commences, until they arrive at

their former positions, and become again impelled by another compression arising from the next vibration. Each of these vibrations produces the same effect amongst the particles adjacent; and in this manner the vibrations are propagated through the atmosphere at the rate of about 1125 feet per second. The experiments of different philosophers develope the following facts:

(i.) The MUSICAL QUALITY of the sound depends on the uniform regularity with which the vibrations of the exciting body succeed one another.

(ii.) The STRENGTH, or degree of LOUDNESS, depends on the extent and consequent violence of those vibrations.

(iii.) The ACUTENESS, or what is more commonly called the PITCH, depends on the velocity, or the comparative rapidity, with which the vibrations succeed one another.

(iv.) Sound is the louder, the greater the condensation of the air through which it is conveyed. It is not easily transmitted out of one medium into another; and cannot be transmitted through a vacuum or perfectly void space.

MUSICAL INTERVALS.

2. Of all sonorous bodies, that of an elastic string has afforded the most precise and accurate views of the different relations in the acuteness and gravity of sounds. It has been proved by mathematical writers, that the small vibrations of the same string are performed uniformly in the same time; and consequently that the same acuteness or pitch always obtains, the degree of loudness of the tone depending entirely on the extent of the vibrations, or the degree of force with which the string is put in motion. Thus, if one of the lower strings of a violoncello, or any other musical string, be caused to vibrate, either with the bow or the finger, and then left to itself, the tone, which is at first powerful, will gradually decline, in consequence of the regular diminution in the extent of the vibrations; but the peculiar pitch of the note will not change, because the subsequent vibrations, though less in their extent, are all of them performed with the same rapidity of succession.

It is here intended to examine more particularly into the various relations and intervals between such tones as shall constitute natural melody, without any regard to the degree of loudness or the several lengths of time with which they may be continued.

3. The most simple principle is, that the quicker the vibrations, the acuter the tone; and, on this ground, a stretched string will produce a more acute tone in proportion as it is smaller, tighter, or shorter; but our inquiries will be much simplified if we abandon all distinctions relative to thickness and tension, by confining our considerations to strings differing only in length. The different tones will thus be reduced to actual measurement. It seems to be this principle which first led musicians to the consideration of a monochord.*

4. Intervals of music are naturally divided into two kinds, concordant and discordant. The interval or distance between two notes is said to be a *concordant interval,* when, heard together,

---

* A stretched string, under which is placed a moveable bridge, for the purpose of varying its length, so as to produce the several notes.

they coalesce in the ear at short and regular periods, and produce an agreeable effect; but, on the contrary, it is said to be a *discordant interval* when the two sounds, instead of uniting, seem to repel each other, produce two separate and confused sounds, and create a grating sensation in the ear.

5. To every melody, or succession of sounds pleasing to the ear, there is one principal note called the *key* or *ground note,* upon which the relations of all the several sounds depend. This note is always particularly distinguished by the ear, which is seldom satisfied unless it terminates the piece. Our present object is to ascertain the disposition of any musical series of notes in relation to that of their key.

6. All tones which form concordant intervals with the *key-note,* are called *consonances;* and those which form discordant intervals with the *key,* are called *dissonances.*

7. The vibrations being the cause of the sounds, the coincidences of those vibrations are found to attend their concordance. For instance, if there be two strings A, B, such that the latter shall perform exactly twice as many vibrations as the former, in the same time, and

we suppose them to commence vibrating together, there will be for every vibration of A two of B; and, therefore, whenever A has completed a vibration, B will have exactly completed a second vibration; and in connexion with this successive coincidence, the two sounds performed together form a concord. These concords are generally found to be the more agreeable as the coincidences are more frequent and less interrupted.

8. It is known, from mathematical investigation, that the numbers of vibrations performed in the same time by strings, differing in length only, are proportional to their lengths taken inversely. Thus, in order that a string B shall vibrate twice as fast as another string A, it must be only half the length of A. Similarly, if A performs 3 vibrations while B performs 4, the lengths of A and B will be as the numbers 3, 4, taken inversely, or as 4 to 3; or, which amounts to the same, B will be $\frac{3}{4}$ths of the length of A; and, in this case, the vibrations will coincide at every 3 vibrations of A and 4 of B.

9. Here it must be carefully observed, that, as the peculiar pitch of a note depends on the

rapidity of the vibrations of the string by which it is produced, so the distance between two notes in acuteness depends solely on the ratio or quotient of the vibrations, or, which is the same, the quotient of their lengths; this quotient is usually taken to represent the interval. For example, if the string B be exactly $\frac{3}{4}$ths of the length of A, the corresponding notes will have the same interval or distance, whatever be the particular lengths of the strings. If A be eight inches and B six inches, or A twelve inches and B nine inches, the interval will be precisely the same, although the two tones in the former case are both of them higher in pitch on account of shorter strings. The interval is, hence, very properly called the interval $\frac{3}{4}$; and all intervals are designated in a similar manner.

10. The interval $\frac{1}{2}$, or the interval between the tones produced by two strings, A and B, when the latter is half the length of the former, is the interval of an octave; and the latter note is called the octave of the former, being the eighth note from it, counting upwards, that is, including the first, which is always done in music.

Of the most perfect coincidence of any note with its octave, either above or below, every person is sensible; when performed together they seem as if but one sound, and, in many cases, require a good practised ear to distinguish the one from the other. In consequence of this minute agreement, musicians give octaves the same names as the original notes.

11. To avoid fractions, let us assume 360 for the length of the string which sounds the key-note of any series. Then is 180 the length, answering to the octave ascending. It has been found by experience, that proportions exceeding the number 5 are generally discordant, as the coincidences become then so very seldom. Our consonances would be thus limited to the proportions—

$$\frac{4}{5}, \frac{3}{4}, \frac{2}{3}, \frac{3}{5}, \frac{1}{2}.$$

It has also been observed, however, that each note which forms a concord with the key-note, also forms a concord with its octave. Now, since the interval from $\frac{4}{5}$ to the octave $\frac{1}{2}$ is determined by the proportion of $\frac{4}{5}$ to $\frac{1}{2}$, or of 8 to 5; it may, like the rest, be expressed by the fraction $\frac{5}{8}$. Similarly the distances of the other three consonances $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, from the octave, are as under:

The octave $\frac{1}{2}$ is above $\frac{3}{4}$, by the interval $\frac{2}{3}$
... ... $\frac{2}{3}$ ... $\frac{3}{4}$
... ... $\frac{3}{5}$ ... $\frac{5}{6}$.

These four intervals furnish only two consonances in addition to the former, which already contain $\frac{2}{3}$ and $\frac{3}{4}$.

We have, therefore, seven simple consonances to any key-note in music, which, arranged with their proper lengths and names, are as follow:

Proportions.	1	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{3}{5}$	$\frac{1}{2}$
Names.	Key-note, or First.	Third minor.	Third major.	Fourth.	Fifth.	Sixth minor.	Sixth major.	Octave, or Key.
Lengths of Strings.	360	300	288	270	240	225	216	180

In this series, the third-minor and sixth-minor properly belong to what, in practice, is called the minor-mode or minor-key. The others, all of which belong to the major-key, stand thus:

Proportions.	1	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{1}{2}$
Names.	Key note, or First.	Third major.	Fourth.	Fifth.	Sixth major.	Octave, or Eighth.
Lengths of Strings.	360	288	270	240	216	180

It will here be observed, that the length of the string which sounds the octave, is just ¾ths of that for the fifth, and consequently that the interval between the fifth and octave is exactly the same as the distance of the fourth from the key-note. The whole is, therefore, by the line which is drawn, divided into two equal parts, which, by the ancients, were called tetrachords; and, to make them complete, another note is inserted in each, viz. one in the former, answering to the interval between the fourth and fifth, and one in the latter, answering to the interval between the key-note and the third-major.

These are two auxiliary notes, and are simultaneously discordant with the key-note, their use being to equalise, as much as possible, the disposition of the series, and to form concords with each other, and also with some of the other notes. The following, then, is the complete diatonic, or natural series, throughout an octave, when the key of a melody is major.

Proportions.	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$
Names.	Key-note, or First.	Second.	Third.	Fourth.	Fifth.	Sixth.	Seventh.	Octave, or Eighth.
Lengths of Strings.	360	320	288	270	240	216	192	180

In this scale, the string which sounds the key or principal note, may be taken of any length, and by dividing it into 360 equal parts, the figures will represent the numbers of these parts contained in the lengths, which produce the successive sounds as they proceed from the principal.

12. But, although these different lengths of strings sufficiently identify the several sounds as related to that of the key, they do not present any immediate measure of their precise distances, in acuteness, from each other, or from the key-note. In order to determine these, we must have recourse to our original principle (art. 9), that two sounds preserve exactly the

same distance, or interval, as long as the lengths of the strings have the same proportion to each other; and the operations will perhaps be rendered more generally intelligible if we premise the octave to be divided into 12 equal parts. Thus, if 12 successive ascending intervals, all exactly alike, terminate in the octave of the first note, and if we can find how many of these are contained in any other interval, we shall have a very good invariable standard for the measurement of all intervals whatever. These equal intervals, 12 of which constitute an octave, we shall call *mean semitones.*

For distinction, let the length of the string which sounds the key-note be denoted by k; the length of the string which sounds a note *one* mean semitone above the key-note by s_1; that which sounds a note *two* mean semitones from the key by $s_2 \ldots\ldots$; and, of course, the octave string will be denoted by s_{12}. Also let r denote the fraction which determines the interval of a mean semitone, or, which is the same thing, let the string s_1 be $k \times r$. Then, as the strings $k, s_1, s_2, s_3 \ldots\ldots\ldots s_{12}$ succeed each other by equal intervals, we shall have, according to the principle just mentioned, $s_1 = kr$, $s_2 = s_1 r$,

$s_3 = s_2\, r$, &c., and $s_{12} = s_{11}\, r$. These relations, expressed by logarithms, are

$$\log s_1 = \log k + \log r = \log k - \log\frac{1}{r} \ldots\ldots (1),$$

$$\log s_2 = \log s_1 - \log\frac{1}{r} = \log k - 2\log\frac{1}{r} \ldots\ldots (2),$$

$$\log s_3 = \log s_2 - \log\frac{1}{r} = \log k - 3\log\frac{1}{r} \ldots\ldots (3),$$

&c. &c. &c.

$$\log s_{12} = \log s_{11} - \log\frac{1}{r} = \log k - 12\log\frac{1}{r} \ldots (12).$$

Thus, we see that the logarithms of the lengths of this series of strings decrease regularly by the quantity $\log\frac{1}{r}$, which we have used instead of $\log r$, because the value of $\log r$, in consequence of r being less than unity, is negative. It will similarly appear from the above, that the logarithm of the length of a string which corresponds with a note *half* a mean semitone from the key, is $\log k - \frac{1}{2}\log\frac{1}{r}$, found by diminishing $\log k$ by *half* of $\log\frac{1}{r}$; and that the logarithm of a string corresponding to any number, n, of mean semitones above the key-note, is $\log k - n\log\frac{1}{r}$, found by diminishing $\log k$ by n times $\log \frac{1}{r}$, whether n be whole or fractional. It follows, therefore, that the number (n) of mean semitones between the key-note and another note produced by any string whose length is s, is determined by the formula

$$n = \frac{\log k - \log s}{\log \frac{1}{r}},$$

and that the number (n') of mean semitones between two sounds produced by any two strings, s, s', is

$$n' = \frac{\log s \sim \log s'}{\log \frac{1}{r}}.$$

To compute such an interval, it is necessary to know the value of $\log \frac{1}{r}$, which may be found from the last of the above logarithmic relations, marked (12). Thus, since the octave string s_{12} is just half of that of the key, their relation, expressed logarithmically, is $\log s_{12} = \log k - \log 2$; and, by comparing this with (12), we see that $12 \log \frac{1}{r}$ must be equal to log 2, or, by using the common logarithms, that

$$\log\frac{1}{r} = \frac{1}{12}\log 2 = \cdot 0250858.$$

This subtracted successively from the logarithm of 360, we get the logarithms of the several strings; and by taking out the corresponding natural numbers, the several lengths come out as hereunder arranged, with the relative places of the diatonic notes (11).

$k = 360\cdot000$	360 = key-note.
$s_1 = 339\cdot795$	
$s_2 = 320\cdot724$	320 = second.
$s_3 = 302\cdot723$	288 = major-third.
$s_4 = 285\cdot732$	
$s_5 = 269\cdot695$	270 = fourth.
$s_6 = 254\cdot558$	
$s_7 = 240\cdot271$	240 = fifth.
$s_8 = 226\cdot786$	216 = major-sixth.
$s_9 = 214\cdot057$	
$s_{10} = 202\cdot043$	192 = seventh.
$s_{11} = 190\cdot703$	
$s_{12} = 180\cdot000$	180 = octave.

13. The disposition of the diatonic notes amongst this series of equal semitones is here distinctly seen, and we might determine their

several intervals in terms of mean semitones by interpolation; but it will be much better to find these by dividing the differences of their logarithms by the value of $\log \frac{1}{r}$, according to the preceding formula; thus, if n be the number of mean semitones from the key-note to another produced by a string whose length is s, the number n is (12), determined by

$$n = \frac{\log k - \log s}{\cdot 0250858}.$$

14. By using 360 for k, and the other numbers severally for s, we get the number of mean semitones in each of the intervals of the diatonic series ascending from the key-note, as in the following arrangement:

	Interval in mean semitones.	Consecutive intervals.
Key-note from Key-note	0·0000	
		2·0391
2d	2·0391	
		1·8240
3d	3·8631	
		1·1173
4th	4·9804	
		2·0391
5th	7·0195	
		1·8241
6th	8·8436	
		2·0391
7th	10·8827	
		1·1173
Octave or 8th	12·0000	

The right-hand column shows the interval between each note and the succeeding one, and is formed by the differences of the other numbers.

It here appears that, in the gradations of the musical scale, there are only three different elemental intervals, viz. 2·0391, 1·8240, and 1·1173. The first of these intervals musicians call a *major-tone,* the second a *minor-tone,* and the third a *limma;* the ratios of these intervals being $\frac{8}{9}$, $\frac{9}{10}$, and $\frac{15}{16}$ respectively. By taking the difference between a *major* and *minor-tone,* a very small interval is found, which is called a *comma,* and is 0·2151 parts of a mean semitone. The ratio corresponding to a comma is $\frac{80}{81}$, because the minor and major-tones $\frac{8}{9}$ and $\frac{9}{10}$ have that ratio, one to the other. We shall here state these intervals distinctly:

	Ratio.	Interval in mean semitones.	Character.
Major-tone....	$\frac{8}{9}$	2·039100017308	.... t
Minor-tone....	$\frac{9}{10}$	1·824037121341	.... $t_{,}$
Limma	$\frac{15}{16}$	1·117312852698	.... θ
Comma	$\frac{80}{81}$	0·215062895967	.... c.

15. These are expressed in mean semitones,

twelve of which constitute the octave. Now, it will be useful to divide the octave into such a number of equal divisions that each interval of the scale may comprise an integral number of them, as these whole numbers will be more convenient for usual practical purposes than the decimal values in mean semitones. This purpose will be answered if the number of divisions representing the octave be such as will render the major and minor-tones and limma whole numbers, since all other intervals result from the various combinations of these elemental ones.

Divide the foregoing values by 12, and we get

$$\left.\begin{array}{lcl} \textit{Major-tone} & = & \cdot169925001442 \\ \textit{Minor-tone} & = & \cdot152003093445 \\ \textit{Limma} & = & \cdot093109404391 \end{array}\right\} \textit{Parts of the Octave.}$$

Now, by multiplying each of these by 730, the results are 124·045, 110·96, and 67·97, which may be regarded as 124, 111, and 68; the greatest error of such suppositions being that of the first, viz. $+\cdot045$, or $\frac{\cdot045}{730} = \cdot00006$ parts of the octave, or about $\frac{1}{2700}$th of a tone.

Hence, by dividing the octave into 730 equal intervals, which we shall call *degrees*, the elemental intervals will be

Major-tone, $t = 124$
Minor-tone, $t_{,} = 111$
Limma, $\theta = 68$
Comma, $c = 13$
Degrees.

These numbers present a more accurate measurement of the musical scale than any other, unless we go to very high numbers. The greatest error which can arise from their natural or melodious combinations is that of the fifth, and does not amount to one half of the error of the major-tone above mentioned.

The concordant intervals are

Minor-third	192	*Degrees.*
Major-third	235	
Fourth	303	
Fifth	427	
Minor-sixth	495	
Major-sixth	538	
Octave	730	

Integral representations of the scale have been given by different authors. It has been proposed by some to divide the octave into 53 divisions, taking 9 of them for the major-tone, 8 for the minor-tone, and 5 for the limma, which furnishes a pretty accurate scale. Others have supposed the periphery of a circle to denote the octave, and reduced the various intervals to corresponding degrees, minutes, and seconds of the circumference; which division, however, although neatly emblematic of the return of the octave, can be of no advantage in musical computations.

Sir J. Herschel, in his extensive and interesting paper on Sound, vol. II. of mixed Sciences, Encyclopedia Metropolitana, has divided the octave into 301 parts, and taken 51 for the major-tone, 46 for the minor-tone, and 28 for the limma. If a tone be divided into 10000 equal parts, the errors, expressed in numbers of these parts, of the diatonic series of notes according to each of the three divisions, will be nearly as in the following table:

	301 Division.	53 Division.	730 Division.
Error of Key	0	0	0
... 2d	− 29	− 7	− 4
... 3d	+ 20	− 70	− 1
... 4th	+ 15	+ 3	+ 2
... 5th	− 15	− 3	− 2
... 6th	+ 35	− 67	+ 1
... 7th	+ 5	− 74	− 2
... 8th	0	0	0

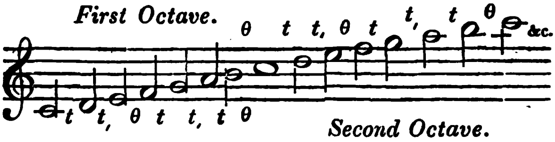
These errors are too insignificant to render it of any importance which method of division is used in common calculations. The last, which has been found by various trials, is that which differs less than any other from the true series, unless we ascend to very high numbers; and is the one which is therefore most to be recommended. According to this division of the octave into 730 degrees, which we shall hereafter adopt, the diatonic scale will be—

Key to	Key ...	0			
			... 124	... t ...	Major-tone.
	2d ...	124			
			... 111	... $t_,$ ...	Minor-tone.
	3d ...	235			
			... 68	... θ ...	Limma.
	4th ...	303			
			... 124	... t ...	Major-tone.
	5th ...	427			
			... 111	... $t_,$ ...	Minor-tone.
	6th ...	538			
			... 124	... t ...	Major-tone.
	7th ...	662			
			... 68	... θ ...	Limma.
	8th ...	730			

16. On the right, we see the several gradations from each note to the succeeding one, which occur in the following order:

Key, or 1st 2d 3d 4th 5th 6th 7th 8th.
t $t_,$ θ t $t_,$ t θ

The tones t, $t_,$ and the limma θ, are thus placed in such a manner that no two consecutive ones are alike; and if we proceed on through the next octave, they must all recur in exactly the same order. Thus, if we take C for the fundamental note, which musicians have assumed for that of the natural key, the continued series will be



in which the second octave follows the same order of succession as the first; and similarly through all other octaves.* In the formation of this scale, the notes E, F, G and A, which are, in relation to the key-note C, the *third-major*, *fourth*, *fifth*, and *sixth-major*, were determined by their *concordance* with the fundamental note C. But D and B, the *second* and *seventh*, which are *discordant* with the key-note, are auxiliaries introduced for the purpose of equalizing the series (see page 10), and were taken so as to make the two tetrachords C D E F, G A B C, complete.

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* It is supposed that all the notes which can be distinguished by the human ear, are within the limits of about nine octaves.

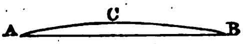
As this assumption is quite arbitrary, we may, at any time, take the liberty of a small alteration in the note D, by making the minor-tone $t_{,}$ from C to D, and the major-tone t from D to E; which is, in effect, merely an interchange of these intervals, and will not interfere with any of the other notes. The former D will correspond with that second, which answers to the natural harmonic of a string when it vibrates in 9 divisions (see page 30): the latter will make the two tetrachords C D E F, G A B C, exactly alike in their intervals.

The other auxiliary B, however, cannot be altered in the same way without violating the melody of the series, as the two adjacent intervals, t and θ, are too unlike to admit of such a transposition.

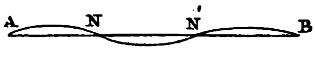
According to the preceding series, the third-minor from A to C ascending is perfect, and contains the major-tone t and the limma θ; whilst that from D to F is imperfect, being formed with the minor-tone $t_{,}$ instead of the major. This latter third would be rendered perfect by the alteration in D we have just mentioned, which would, however, at the same time destroy the perfection of the third B D. The most natural method in actual performance to satisfy the ear would be, always to introduce that D which forms a true concord with the principal note that immediately precedes it. But this difference in the note D, which in theory is a comma, is entirely done away with in practice, as the harmonic advantages which could be derived from the true theoretical scale, as directed by nature, would by no means compensate the difficulties of its performance. Musicians do not, and, according to the construction of instruments, cannot follow the true series, but are obliged to adopt artificial scales, which approach as near to the true as the various difficulties will allow, and which will be explained under the article TEMPERAMENT.

HARMONICS.

17. We have determined our various intervals by means of the common properties of the musical string, without any consideration of the manner in which it vibrates. Now, it is known both by theory and experiment, that a stretched cord may vibrate in many different ways. The *first* and most simple is, when, at every instant it assumes the figure of a regular harmonic curve, A C B, without any change in its flexure, as shewn in the annexed figure. These vibrations are those which produce the FUNDAMENTAL note of the string.



A *second* mode of vibration which may take place, is when the string divides itself into a number of equal parts, and each adjacent two vibrate in opposite directions. Suppose it to divide into three parts, at N and N′; then, as these three divisions vibrate in the same time and in opposite directions, it is



evident that the points N, N′, which are called *nodes*, are always kept at rest by equal tensions in opposite directions, and, consequently, that such a character of vibration, when once communicated, will, if not interrupted, be sustained by the string until its motion ceases. The tone produced by these vibrations cannot correspond with that of the former, but will be the same as if produced by the vibrations of the three divisions A N, N N′, N′ B, considered as separate strings, each of which is one third of the original in length. All tones which arise from this kind of vibrations, are called HARMONICS to the FUNDAMENTAL note of the string.

A string very seldom vibrates exclusively either to one or other of these modes of vibration, but generally partakes of both, and often consists of several different modes all in action together. For when the string is vibrating wholly, and producing its fundamental note, it is generally subdivided into various portions, each of which is vibrating at the same time, and producing a harmonic. The mathematical theory of the motion of a musical string shows that any number of different kinds of vibrations, which can be communicated and sustained separately, may be communicated and sustained altogether at the same time; and, hence, we see the reason why the fundamental notes of large strings, such as those of the harpsichord and violoncello, are usually accompanied with harmonic notes, which are more or less sensible, according to the strength or weakness of the vibratory agitation of the portions into which the string has divided itself; they are most readily communicated by a sudden action on the string near to one of its extremities, and, therefore, almost always accompany the tones of the pianoforte, particularly those of the bass. These harmonic sounds are powerfully heard in the sound of bells, and can also be produced on wind instruments, such as Trumpets, French horns, Organ pipes, &c., by varying the force of injected air; and on the German flute, by different degrees of contraction in the lips.

18. Harmonic notes are sometimes intentionally introduced into performances by violin players, who produce them by means of a slight pressure of the finger on the string of the instrument, at a proper point for aliquot division. Thus, if, in the latter of the two fore-

going figures, A B be supposed to represent a string of the violin, which passes over the nut at A and the bridge at B; then if the performer, whilst drawing the bow between N′ and B, presses lightly the point N of the string with his little finger, the string will, instead of vibrating wholly between A and B, divide at the other point N′, and vibrate as if it were three different equal strings, A N, N N′, N′ B, which will unite in producing a *harmonic note*, unaccompanied by the fundamental.

Now, A N′ being *two-thirds* of the string, or the length which would sound the fifth, and each division being one half of this length, the harmonic note, thus generated, will be an octave higher than the *fifth* of the *fundamental note;* that is, it will correspond with the fifth in the second octave.

If the bow be made to pass over the point N′, where the string is naturally inclined to divide itself, this mode of vibration which it is disposed to assume will be interrupted, and the result will, in consequence, be a harsh grumbling noise.

We ought here to observe, that this same harmonic will be caused by touching the string with the finger at N′ instead of N, as the string will then divide again at N, and vibrate in precisely the same manner.

19. These tones are distinguished by a peculiar character of melodious softness, which may be accounted for by the want of that rigidity at the nodal points, which exists at the extremities of strings firmly fixed—the positions of these nodal points, or of the extremities of the vibrating divisions, are simply modified by the natural counteraction of directly opposite tensions; but the ends of strings, when producing the usual notes, are so tenaciously stopped as not to admit of the slightest degree of motion.

20. As all harmonic notes arise from the vibrations of the various aliquot parts of the string, they are wholly comprised in the ratios $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. In general, when a string is lightly touched at any point, and put in motion with the bow, it will divide itself into the least possible number of equal parts, so that the point where it is touched shall answer to one of the points of division, unless the bow falls on one of the other points where the string will be disposed to divide itself, which would neutralize the effect. If it be touched at the distances $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. of its length, it will be

made to vibrate according to 2, 3, 4, 5, &c. divisions, and the corresponding harmonic sounds, in reference to the fundamental one, will be as below·

Number of Divisions.	Harmonic, in unison with †
2	Octave.
3	Fifth, in second octave.
4	Second octave.
5	Major third, in third octave.
6	Fifth.
* 7	Flat seventh.
8	Third octave.
9	Second, in fourth octave.
10	Major third, in fourth octave.
* 11	Fourth.
12	Fifth.
* 13	Sixth.
* 14	Flat seventh.
15	Seventh.
16	Fourth octave.
* 17	Flat second, in fifth octave.
18	Second.
* 19	Flat third.
20	Major third.
&c.	&c.

---

† This is the precise order in which the notes are produced on the Trumpet and French horn, when sounded gradually upwards. The five first may be produced from the lower notes C, C♯, D, E♭ of the German flute, by the action of the lips.

These all perfectly agree, as expressed, with the corresponding tones in the true diatonic scale, except those marked with an asterisk, which are as follow:

Divisions.		Pitch.
7	..	2 octaves and 589 degrees.
11	..	3 .. 335 degrees.
13	..	3 .. 511 degrees.
14	..	3 .. 589 degrees.
17	..	4 .. 64 degrees.
19	..	4 .. 181 degrees.

We here see that the variations of these harmonics from the above list are,

Divisions.		Pitch.
7	...	17 degrees too flat.
11	...	32 degrees too sharp.
13	...	27 degrees too flat.
14	...	17 degrees too flat.
17	...	4 degrees too flat.
19	...	11 degrees too flat.

21. It may be useful here to reduce the expression we have already given (art. 13) for the computation of the interval from the key to any other note in terms of mean semitones, to another for a direct computation in *degrees* according to our division of the octave into 730. This is simply done by dividing by 12, which

will give the interval in parts of the octave, and then multiplying by 730. We hence find the number of degrees in the interval between the key-note, the length of whose string is k, and another, the length of whose string is s, to be determined thus:

$$\textit{Number of degrees} = \frac{\log k - \log s}{\cdot 00041237}$$

$$= \quad 2425\,(\log k - \log s) \; = \frac{9700}{4}\,(\log k - \log s)$$

$$= \left\{ \frac{10000\,(\log k - \log s)}{4} \right\} - \frac{3}{100} \left\{ \frac{10000\,(\log k - \log s)}{4} \right\}.$$

Hence, when a harmonic is produced by the string of the key-note, dividing itself into a number (n) of equal divisions, each of these divisions, or the length which determines the acuteness of the harmonic being $\frac{k}{n}$, its distance or interval from the fundamental note of the string is,

$$\textit{in degrees} = \frac{\log n}{\cdot 00041237}$$

$$= \left\{ \frac{10000 \log n}{4} \right\} - \frac{3}{100} \left\{ \frac{10000 \log n}{4} \right\}.$$

The preceding results may serve as examples for any one who may be inclined to amuse himself with these harmonic calculations, in which five places of logarithms will be sufficient. (See pages 30, 31.)

22. In concluding these brief remarks on this very interesting part of our subject, we shall add a list of the numbers of vibrating divisions of a string of the violin, or any other musical string, according to the fingerings of the various notes in ascending the scale, of which the fundamental note of the string is the key-note. With this and the preceding list of harmonic divisions we can readily see the particular note which is in unison with the resulting harmonic. No finger touches the string except that one which answers to the note, and we might begin at the octave or at the middle point of the string, because harmonics come most readily out from a fingering near to the bridge, although the same precise notes may be produced by a fingering at the same distance from the nut.

In the *first* octave, beginning with the *open* string, the divisions are

Note...... ...Key	2d	3d	4th	5th	6th	7th	Octave.
Divisions ... 1	9	5	4	3	5	15	2

♭ 3d .. 6 divisions.

In the *second* octave, beginning at *half* of the string from the bridge, the divisions are

Note	Key	2d	3d	4th	5th	6th	7th	Octave
Divisions ...	2	9	5	8	3	10	15	4.
	×				×			×

In the *third* octave, beginning at *one fourth* of the string from the bridge, the divisions are

Note	Key	2d	3d	4th	5th	6th	7th	Octave
Divisions ...	4	9	5	16	6	20	15	8.
			×		×			×

In the *fourth* octave, beginning at *one eighth* of the string from the bridge, the divisions are

Note	Key	2d	3d	4th	5th	6th	7th	Octave
Divisions ...	8	9	10	32*	12	40*	15	16.
	×	×	×		×		×	×

In this series, all the numbers we have before marked as imperfect are excluded, and those to which a × is annexed, are such as succeed each other with a regular progression of acuteness.

The musical sounds produced in this manner with the slight contact of *one* finger, which are called *natural harmonics,* are not the only ones which may be played on the violin; others can be produced by stopping firmly with the first or fore finger, when the hand is in any position, and touching the string lightly with one of the other fingers: these are called *artificial harmonics,* and can be so varied as to render this first of instruments capable of affording a complete scale of harmonic sounds.

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* Too acute to be produced and heard on the small violin.

TEMPERAMENT.

23. The sound of the key-note is readily distinguished from the others by the particular relation in which it is placed in the scale. This fundamental note is that from which the pitches of all the others have their origin, and the ear always dwells upon it with satisfaction whenever it occurs either in melody or in harmony. If, from any assumed musical sound whatever, a series be formed, ascending or descending, which succeed each other in the same order of intervals as those of the diatonic scale from the fundamental note, that original sound, and all its octaves which may occur, will at once be recognized and received by the ear as the key-note of the series, or of any melody which may involve them. It is therefore desirable, for the purpose of variety, as well as of convenience in a choice of pitch, that the musical scale be so contrived as to comprise a succession of sounds which will yield a very approximate diatonic series on the assumption

of any one of them as a key-note. The practical formation of such a scale will impose the necessity of changing most of the notes from their true musical pitch by an interval so minute as not to offend the ear; this is called tempering the scale, and any note or interval is said to be tempered sharp or flat, according as it is elevated or depressed, increased or diminished :—all intervals may be slightly tempered except octaves, which must necessarily be tuned perfect.

24. Amongst the various systems of temperament which have been devised, the most simple one, which is improperly called the vulgar temperament, consists in dividing the octave into 12 equal intervals, corresponding with our mean semitones; and any one of these 12 sounds being taken as the key-note, the intervals in ascending an octave will be—

	2		2		1		2		2		2		1		...... semitones
Key	⋮	2d	⋮	3d	⋮	4th	⋮	5th	⋮	6th	⋮	7th	⋮	8th	
	A tone		A tone		A semitone		A tone		A tone		A tone		A semitone		

In this scale of *equal semitones*, the greatest imperfections are those of the 3d and 6th, which are respectively too sharp, about 8 and 9½ *degrees* of the 730 division of the octave.

The temperaments are—

Minor-third	9·5151 *degrees*	*flatter*	than perfect.
Major-third	8·3258	*sharper*	
Fourth	1·1893	*sharper*	
Fifth	1·1893	*flatter*	
Minor-sixth	8·3258	*flatter*	
Major-sixth	9·5151	*sharper*	

These deviations, however, which are considerably less than a comma or 13 degrees, are too small to affect, in a very sensible degree, the melody of the intervals.

This method of temperament bears with it the recommendation that all keys are rendered equally perfect, and none of them very perceptibly imperfect. Some musicians notwithstanding object to it, on the ground that it does not possess a single chord or interval that is really perfect. In reply to this, it may be observed that no interval can be made absolutely perfect without deteriorating from the others, and that even an experienced tuner, whose ear is his only guide, is liable to commit similar deviations in an attempt to conform to any system of temperament.

The twelve sounds which compose the octave according to this system, beginning at C, are thus named:

1	2	3	4	5	6	7	8	9	10	11	12	
C	C♯	D	D♯	E	F	F♯	G	G♯	A	A♯	B	C, &c.
	or		or			or		or		or		
	D♭		E♭			G♭		A♭		B♭		

and the same in all octaves.

If these semitones could be tuned perfectly equal, all the different keys would vary only in pitch; none would possess any strong peculiarity distinct from the others, as must be the case when these intervals are slightly unequal. This would form an objection to the temperament by equal or mean semitones, as it is conceived by some musicians that the ear is better pleased with the peculiar differences in the effect and character of the various keys when their respective intervals do not minutely correspond; but still it would not be judicious to reject at once the system under such an uncertain idea, since, even admitting it to be correct, it must be generally acknowledged that no tuner can preserve this exact equality, which he might wish to avoid.

According to this system of temperament, the fifth contains *seven* mean semitones, and, therefore, *twelve* of them will just occupy *seven* octaves. Now, an octave containing 730 *degrees*, seven octaves will make 5110 *degrees*, and, as this must correspond with twelve fifths, each fifth must contain 425⅚ degrees. Thus, we see that the fifths are tempered only about one degree flat, and are therefore very nearly perfect.

25. The scale is hence easily tuned by ascending successively twelve fifths, observing always to take the eighth below whenever we get out of the octave we are tuning. If we begin at C, we shall thus get the series:

C, G, D, A, E, B, F♯, C♯, G♯, D♯, A♯, F, C,

which comprises all the twelve semitones of the octave, the *sharps* answering also for the *flats* of the adjacent upper notes. In tuning these fifths two or three of them must be made rather flat, in order to remove the excess of 14 *degrees*, which twelve ascending *perfect* fifths would have over the seven octaves we descend*; and this will be known by the unison of the last C with the original one, or rather by the harmony of the last fifth F C, using the derived F and the original C.

---

* This excess is, by tuners, called "the wolf."

The above series may evidently be tuned in a retrograde order by successively *descending* a fifth, and taking the octave above when necessary. It is perhaps preferable to tune the first half of them by ascending fifths, and the other by descending fifths, from the primitive note C. Thus we get,

Ascending by fifths ......C, G, D, A, E, B, F♯;
Descending by fifths ...C, F, A♯, D♯, G♯, C♯, F♯;

and this is, in fact, the method generally practised by musicians.*

This scale is, without doubt, the best one for such instruments as the common pianoforte, organ, &c. which must necessarily have but one sound for both a sharp and the flat of the next upper note.

26. A system of sounds may, however, be formed, in which all the keys are more nearly approximate than that of equal semitones, by supposing the diatonic semitone not to be ex-

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* Much valuable practical information on the subject of tuning the pianoforte, is to be found in a neat and explanatory Essay by J. Jousse, Professor of Music, recently published.

actly half of the tone, and consequently not to divide the tone into equal parts. In such a system the diatonic semitone must be greater than half of the tone, because the limma is so in the scale of true intervals; it is in consequence called a MAJOR OR DIATONIC SEMITONE, and the remaining interval of the tone is called a MINOR OR CHROMATIC SEMITONE. The difference of these two semitones constitutes an interval between the sharp of any note and the flat of the one above it, which is called the ENHARMONIC DIESIS; and the scale itself is called the ENHARMONIC SCALE.

It may be interesting here to ascertain the particular values which must be assigned to the *tone* and *diatonic semitone,* so that all the concords shall be affected with the least possible imperfections; and this we shall effect by the principle of least squares. We must first observe, that the *third-minor, third-major* and *fourth,* are the only concords necessary to be considered, because the others are merely the inversions of these, and we know that any error which may increase or diminish a concord, will have precisely the same effect in decreasing or increasing its inversion, as the octave, which is composed of them both, is unchangeable. Let the required tone, and diatonic semitone, be denoted by τ and σ; and these intervals will be

$$\begin{aligned}
\textit{Minor-third} &= \tau + \sigma \\
\textit{Major-third} &= 2\,\tau \\
\textit{Fourth} &= 2\,\tau + \sigma.
\end{aligned}$$

The *perfect* values, according to the notation marked on page 17, are

$$\begin{aligned}
\textit{Minor-third} &= t + \theta = t_{,} + \theta + c \\
\textit{Major-third} &= t + t_{,} = 2\,t_{,} + c \\
\textit{Fourth} &= t + t_{,} + \theta = 2\,t_{,} + \theta + c,
\end{aligned}$$

which, respectively deducted from the above, we find the errors of these intervals in our assumed system to be,

$$\begin{aligned}
&\textit{Minor-third}\ldots\ (\tau - t_{,}) + (\sigma - \theta) - c \\
&\textit{Major-third}\ldots\ 2\,(\tau - t_{,}) - c \\
&\textit{Fourth}\ldots\ldots\ldots\ 2\,(\tau - t_{,}) + (\sigma - \theta) - c.
\end{aligned}$$

But as the octave, which contains five tones and two diatonic semitones, is to be perfect, we must have τ and σ, so that

$$\begin{aligned}
5\,\tau + 2\sigma &= \textit{true octave} \\
&= 3\,t + 2\,t_{,} + 2\,\theta \\
&= 5\,t_{,} + 2\,\theta + 3\,c;
\end{aligned}$$

and, therefore, taking the difference,

$$5\,(\tau - t_{,}) + 2\,(\sigma - \theta) - 3\,c = 0,$$

which will be complied with by taking

$$\sigma - \theta = -\frac{5}{2}(\tau - t_,) + \frac{3}{2}c.$$

With the substitution of this value of $\sigma - \theta$, the above-mentioned errors become,

Minor-third...	$-\frac{3}{2}(\tau - t_,) + \frac{1}{2}c$
Major-third...	$2(\tau - t_,) - c$
Fourth.........	$-\frac{1}{2}(\tau - t_,) + \frac{1}{2}c.$

To determine the value of $\tau - t_,$, so that the sum of the squares of these three errors shall be the least possible, multiply them by the respective coefficients $-\frac{3}{2}$, $+2$, $-\frac{1}{2}$, and the sum of the products will be

$$\frac{13}{2}(\tau - t_,) - 3c.$$

By putting this $= 0$, we find

$$\tau - t_, = \frac{6}{13}c,$$

which used in the above value of $\sigma - \theta$, gives also

$$\sigma - \theta = \frac{9}{26}c.$$

Hence, the values of the tone τ, and diatonic semitone σ, expressed in degrees, are

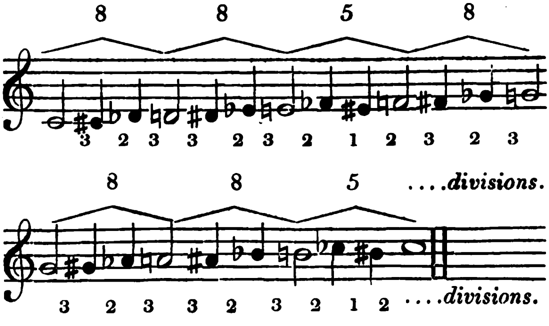
$$\tau = t_{,} + \frac{6}{13}c = 117 \text{ degrees},$$

$$\sigma = \theta + \frac{9}{26}c = 72\tfrac{1}{2} \quad \ldots;$$

and these are very nearly in the ratio of 8 to 5. We may therefore divide the octave into 50 equal divisions, and appropriate 8 of them to the tone and 5 to the diatonic semitone. Thus, the intervals of the diatonic octave will be,

Notes......	Key	2d	3d	4th	5th	6th	7th	8th	
Intervals..		8	8	5	8	8	8	5	...divisions;

And, if we begin at C♮, the complete scale of sounds, which furnishes a series on all the keys, will be—



This system is precisely the same as that which Dr. Smith, in his Treatise on Harmonics,[*] calls the scale of equal harmony. It is decidedly the most perfect of any systems in which the tones are all alike.

The interval of 2 divisions which exists between the sharp of any note and the flat of the one above it, is the *enharmonic diesis.*[†]

Between any note and its sharp or flat there is an interval of 3 *divisions,* which is the *chromatic semitone;* but the intervals between the sharp or flat of a note from the adjacent note, or, which is the same, the interval of a semitone, whose notes have different names, consists of 5 *divisions,* which is the *diatonic semitone.*

The difference between the chromatic semitone and the diesis is an interval of 1 division, which is the *comma.* We may therefore consider the octave as divided into 50 commas.

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[*] Second edition, London, 1759.

[†] Excepting where the diatonic semitones occur, in which cases this interval consists of only one division or a comma.

27. The following is a list of all the major-keys which are necessary in music:

	Key	2d	3d	4th	5th	6th	7th	8th	Intervals.
Natural	(C)	(D)	(E)	(F)	(G)	(A)	(B)	(C)	5 divisions.
Five flats	(D♭)	(E♭)	(F)	(G♭)	(A♭)	(B♭)	(C)	(D♭)	3 divisions.
Two sharps	(D)	(E)	(F♯)	(G)	(A)	(B)	(C♯)	(D)	5 divisions.
Three flats.........	(E♭)	(F)	(G)	(A♭)	(B♭)	(C)	(D)	(E♭)	3 divisions.
Four sharps	(E)	(F♯)	(G♯)	(A)	(B)	(C♯)	(D♯)	(E)	5 divisions.
One flat	(F)	(G)	(A)	(B♭)	(C)	(D)	(E)	(F)	5 divisions.
Six flats	(G♭)	(A♭)	(B♭)	(C♭)	(D♭)	(E♭)	(F)	(G♭)	3 divisions.
One sharp	(G)	(A)	(B)	(C)	(D)	(E)	(F♯)	(G)	5 divisions.
Four flats	(A♭)	(B♭)	(C)	(D♭)	(E♭)	(F)	(G)	(A♭)	3 divisions.
Three sharps......	(A)	(B)	(C♯)	(D)	(E)	(F♯)	(G♯)	(A)	5 divisions.
Two flats	(B♭)	(C)	(D)	(E♭)	(F)	(G)	(A)	(B♭)	3 divisions.
Five sharps	(B)	(C♯)	(D♯)	(E)	(F♯)	(G♯)	(A♯)	(B)	

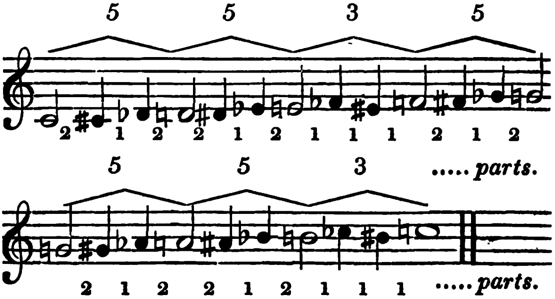
Each of these twelve diatonic series, which are comprised in the foregoing scale, will be found to observe the same succession of intervals, viz. 8, 8, 5, 8, 8, 8, 5; and hence the reason, or rather the necessity, of using the sharps and flats in the several keys to which they belong, and consequently of introducing them to complete the general scale. It will be observed, however, that the sounds E♯, F♭, and B♯, do not enter, and, of course, are not required for any of these keys; they can only be used, therefore, as accidental notes, or for the completion of three other keys, viz. *F♯, six sharps; C♯, seven sharps;* and *C♭, seven flats:* but these additional keys can scarcely be judged necessary, as the key-notes of the series we have already enumerated, supply a regular and complete succession of sounds throughout the octave, as we see by their intervals, as stated on the right-hand of the list. The omission of these keys in musical compositions is a subject well worthy of the attention of musicians.

According to this temperament, all keys are alike in the disposition of their intervals; the values of the *tone* and *major,* or *diatonic semitone* τ, *s,* expressed in *degrees* of the 730 division, are respectively 116·8, and 73; and the errors of the concords, which are the same on all the keys, are

Minor-third	2·2151	*degrees*	too flat.
Major-third	1·4075	...	too flat.
Fourth	3·6226	...	too sharp.
Fifth	3·6226	...	too flat.
Sixth-minor	1·4075	...	too sharp.
Sixth-major	2·2151	...	too sharp.

These variations from true harmony are much too small to be appreciable, being less altogether than in any other system composed of equal tones.

28. We may here mention the system of Huygen's, which has been much approved of by many musicians. He divides the octave into 31 equal parts, and takes 5 for the tone, 3 for the *diatonic semitone*, and, hence, 2 for the *chromatic semitone*, and 1 for the *enharmonic diesis*, so that his scale is as below:



In this scale, like the former, the E♯, F♭, and B♯, are superfluous, and may be omitted, as they do not enter into the usual twelve musical keys before mentioned.

The deviations of the concords of this system, measured in *degrees* of the 730 division, are

Minor-third	3·6280	*degrees*	too flat.
Major-third	0·4764	...	too sharp.
Fourth	3·1517	...	too sharp.
Fifth	3·1517	...	too flat.
Sixth-minor	0·4764	...	too flat.
Sixth-major	3·6280	...	too sharp.

This scale, therefore, has the greatest temperament in the *minor-third,* and its inversion the *major-sixth,* which is the principal objection to it, as it is known that these concords are most readily put out of tune, and consequently should have the least temperament. However, taking all into account, it must be acknowledged to be a very good scale.

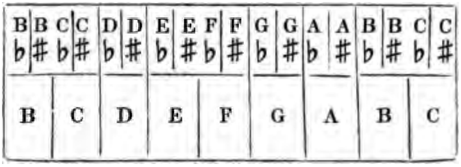
29. For the practical tuning of a keyed instrument, such as the organ, in which the full enharmonic scale is to be introduced, perhaps the best method after all would be to divide the octave into 19 *equal intervals* by 20 keys, which complete it; that is, to make the series,

(C) (C♯) (D♭) (D) (D♯) (E♭) (E) (F♭ or E♯) (F)

(F♯) (G♭) (G) (G♯) (A♭) (A) (A♯) (B♭) (B) (C♭ or B♯) (C),

proceed successively by equal intervals. Such a system would have an advantage similar to that of the scale of mean semitones in the chromatic systems; it would furnish an excellent scale on any one of the 19 sounds as a key-note, and to such perfection as to be perhaps preferable even to those of the two preceding systems, by virtue of the accuracy of the minor-third and major-sixth, and the smallness of the temperaments of the other intervals.

An organ so constructed and tuned would possess one of the principal advantages of perfect instruments, such as the violin and violoncello, viz. a facility of modulation into different keys, and an easy adaptation to a considerable variety of pitch, without any characteristic difference in the mode of fingering. The keys in each octave may be thus conveniently disposed:



Beginning at any key as a fundamental, and including it in the counting, the concords and their temperaments will be,

Minor-third,	(6 keys)...	0·0901	degrees	too sharp.
Major-third,	(7 keys)...	4·4812	...	too flat.
Fourth	(9 keys)...	4·3910	...	too sharp.
Fifth	(12 keys)...	4·3910	...	too flat.
Minor-sixth	(14 keys)...	4·4812	...	too sharp.
Major-sixth	(15 keys)...	0·0901	...	too flat.

Messrs. J. Robson and Son, St. Martin's-lane, have recently built an enharmonic organ, for which the division of the octave into 53 divisions has been adopted, (see page 19.) According to this system, the major tone contains 9, the minor tone 8, and the diatonic semitone 5 divisions; and the temperaments are,

Minor-third......	0·8151	degrees	too sharp.
Major-third......	0·8566	...	too flat.
Fourth	0·0415	...	too sharp.
Fifth...............	0·0415	...	too flat.
Minor-sixth......	0·8566	...	too sharp.
Major-sixth......	0·8151	...	too flat.

which are too minute to be detected by the most critical ear.

The construction of an organ on such a refined scale, in which a distinction is made between the major tone and minor tone, is certainly a bold attempt. To avoid the interference of the comma in some of the concords, double values are given to the dissonances, differing a comma or one division from each other. Thus, on any key, the following series are adopted:

Key.	Minor-second.	Major-second.	Minor-third.	Major-third.	Fourth.	(Tritone.)
0	5, 6	8, 9	14	17	22	26

(Tritone.)	Fifth.	Minor-sixth.	Major-sixth.	Minor-seventh.	Major-seventh.	Octave.
26	31	36	39	44, 45	47, 48	63.

Though the instrument is limited to the most essential notes for the keys from five sharps to four flats inclusive, it must be very complex, as it contains no less than 29 sounds in each octave, viz. 0, 1, 3, 4, 5, 6, 8, 9, 11, 12, 14, 17, 18, 22, 23, 25, 26, 30, 31, 34, 36, 37, 39, 40, 43, 44, 45, 47, 48, (53, octave). To afford a complete series, similar to the above, on each of the 12 keys, page 47, 18 additional sounds would still be required, viz. 10, 13, 15, 16, 19, 20, 21, 27, 28, 32, 33, 35, 38, 41, 42, 49, 50, 52, making in all 47 sounds in each octave; and if the remaining 6 sounds, 2, 7, 24, 29, 46, 51, were added, the instrument, as in the scale of 19 equal intervals, would yield a full series on any one of the 53 sounds. Great credit is due to this undertaking; but it would be too much to expect from it complete success, as the superabundance of keys must prove an insuperable impediment to the execution.

The scale of 19 sounds in the octave, which gives great accuracy to the harmony, though not to such theoretical minuteness as the scale of 53 sounds, would, on account of its simplicity and easy adaptation to the construction of the instrument, be a useful improvement in the correctness of the harmony, without infringing on its practicability as regards the performer. It is to be hoped that some persevering experimental musician will construct an instrument on this scale, and give it a fair trial.

30. For the tuning of instruments, in which accuracy is particularly wanted, and a definite system of temperament is to be adhered to, a monochord is often very convenient for the detection of any suspected error in the intervals. The most inexperienced practitioner may, by its use, follow any proposed scale with great mi-

nuteness. In the practical application of this principle, it will be necessary to have recourse to such a table as the following, page 56, which shows the parts of the monochord for each interval according to the respective divisions of the octave into 12, 19, 31, 50 and 53 equal parts. It has been calculated from the formula,

$$\log s = \log k - f \log 2,$$

in which k represents 360, the length of the whole string, s the length which produces the proposed sound, and f the corresponding fractional part of the octave from the fundamental sound.

It should here be observed, that it will be preferable to have the string stretched by the weight in a vertical position, as it is of consequence to preserve as perfect a uniformity as possible in the tension, which is so liable to be affected by the friction arising from the pressure on the bridge, when placed in a horizontal position. The effect of this friction is more perhaps than would be supposed: each removal of the bridge has a tendency to alter the tension of the string. But this is almost entirely obviated by the vertical position.

12 equal Intervals.	Monochord.	19 equal Intervals.	Monochord.	31 equal Intervals.	Monochord.	50 equal Intervals.	Monochord.	53 equal Intervals.	Monochord.
0	360·00	0	360·00	0	360·00	0	360·00	0	360·00
						1	355·04	1	355·32
				1	352·04	2	350·16	2	350·71
		1	347·10	2	344·26	3	345·34	3	346·15
1	339·79					4	340·58	4	341·65
				3	336·64	5	335·90	5	337·21
		2	334·67			6	331·27	6	332·83
				4	329·20	7	326·71	7	328·51
		3	322·68			8	322·21	8	324·24
2	320·72			5	321·92			9	320 03
				6	314·80	9	317·77	10	315·87
		4	311·12			10	313·40	11	311·76
				7	307·84	11	309·08	12	307·71
3	302·72					12	304·83	13	303·71
		5	299·97	8	301·04	13	300·63	14	299·77
				9	294·38	14	296·49	15	295·87
						15	292·41	16	292·03
		6	289·23	10	287·87	16	288·39	17	288·23
4	285·73					17	284·41	18	284·49
				11	281·50	18	280·50	19	280·79
		7	278·87			19	276·64	20	277·14
				12	275 28	20	272·83	21	273·54
5	269 70	8	268 88	13	269·19	21	269·07	22	269·99
						22	265·37	23	266·48
				14	263·24	23	261·72	24	263·02
		9	259 24			24	258·11	25	259·60
				15	257·42			26	256·23
6	254·56			16	251·73	25	254·56	27	252·90
		10	249·96			26	251·05	28	249·61
				17	246·16	27	247·60	29	246·37
						28	244·19	30	243·17
7	240·27	11	241·00	18	240·72	29	240·83	31	240·01
				19	235·40	30	237·51	32	236·89
		12	232·37			31	234·24	33	233 81
				20	230·19	32	231·02	34	230·78
8	226·79					33	227·84	35	227·78
		13	224·04	21	225·10	34	224·70	36	224·82
						35	221·61	37	221·90
				22	220·12	36	218·56	38	219·01
		14	216·02	23	215 26	37	215·55	39	216·17
9	214·06					38	212·58	40	213·36
				24	210·50	39	209·65	41	210·59
		15	208·28			40	206·77	42	207·85
				25	205·84	41	203·92	43	205·15
10	202·04			26	201·29			44	202·48
		16	200·82			42	201·11	45	199·85
				27	196·84	43	198·34	46	197·26
		17	193·62			44	195·61	47	194·69
				28	192·49	45	192·92	48	192·16
11	190·70					46	190·26	49	189·67
		18	186·69	29	188·23	47	187·64	50	187·20
				30	184·07	48	185·06	51	184·77
						49	182·51	52	182·37
12	180·00	19	180·00	31	180·00	50	180·00	53	180·00

BEATS OF IMPERFECT CONCORDS.

31. Before we conclude, we must notice a very interesting phenomenon which accompanies concords that are nearly, but not accurately, perfect. Such are all tempered concords, which are attended with a series of pulsations of sound, called BEATS; and these are distinctly audible on the lower strings of the harpsicord, but more powerfully so on the organ, whereon the notes can be firmly continued without any diminution of their strength. They originate in the slight perturbation of the periodic coincidences of the vibrations which take place with the perfect chord, which perturbation makes one of them gain a little upon the other at these successive places of coincidence; and as soon as this small gain brings two consecutive vibrations about to another coincidence, a long cycle of the vibrations will be completed, and they will begin and perform a similar long cycle in the same manner, and so on. The coincidences at the

beginning and end of these cycles are the times when the beats are heard, and consequently the time of the cycle is the same as that between two consecutive beats. They seem to be caused by the mutual concurrence of the vibrations in setting the particles of the surrounding air in motion, which makes the joint sound more loud at the places of coincidence, though not with sufficient frequency to produce the sensation of a continued sound. Since, by these beats, the scale of an organ can be tuned more nicely than by any other means, we shall here investigate all particulars concerning them which may be applicable to such a purpose, and which may assist the practical musician in any necessary calculations connected with this subject.

32. Take the lower note as a key-note, and assume k for its length in inches, and N for the number of its vibrations in one second; also let s be the length in inches of the upper string, which forms a *perfect* concord, and let $\frac{m}{n}$ be the fraction which represents the interval of this concord, being $= \frac{s}{k}$, expressed in *least* whole numbers. Then the time of one vibration

of $k = \frac{k}{s}$, or $\frac{n}{m}$, vibrations of s; and, therefore, when

k performs $1, \quad 2, \quad 3, \; \ldots\ldots m$ vibrations,

$s \quad \ldots \quad \frac{n}{m}, \quad 2\frac{n}{m}, \quad 3\frac{n}{m}, \ldots\ldots n \quad \ldots \quad ;$

and, since the numbers m and n are prime to each other, the value in the latter series cannot become *integral* until m vibrations of k, and consequently n vibrations of s, are completed, when a coincidence will therefore again take place. The vibrations will then have finished a small cycle from coincidence to a repeated coincidence, the time of this short cycle being m vibrations of k or $\frac{m}{N}$. This we shall call the *cycle of the concord.*

Again, the m terms of the fractions $\frac{n}{m}$, $2\frac{n}{m}$, $3\frac{n}{m} \ldots\ldots n$, will consist of m whole numbers, connected with the simple fractions $\frac{1}{m}$, $\frac{2}{m}$, $\frac{3}{m} \ldots\ldots \frac{m-1}{m}$, 0, differently arranged; and hence, at the two particular vibrations of k in each cycle, where this simple fraction becomes $\frac{m-1}{m}$ and $\frac{1}{m}$, the string s will be just $\frac{1}{m}$th of its own vibration behind and in advance of that of k, and these will evidently be the most contiguous vibrations of the two strings which occur in the cycle of the concord.

Suppose the upper string s to be now a little tempered, either sharp or flat, by varying its length to s', and the number of its vibrations will then be multiplied by $\frac{s}{s'}$, which will be very nearly unity; so that when

$$\begin{array}{llllll} k \text{ performs} & & 1, & 2, & 3, & \ldots\ldots m \text{ vibrations.} \\ s' & \ldots & \frac{n}{m}\cdot\frac{s}{s'}, & 2\,\frac{n}{m}\cdot\frac{s}{s'}, & 3\,\frac{n}{m}\cdot\frac{s}{s'} & \ldots\ldots n\,\frac{s}{s'}, \quad \ldots \end{array}$$

At the end of the cycle of the concord s' will, therefore, have performed $n\,\frac{s}{s'}$ vibrations instead of n vibrations, and will have gained or lost the part of a vibration expressed by $n\left(\frac{s}{s'} \sim 1\right)$, which destroys the coincidence. It is evident that it will continue in the same manner to gain or lose this quantity in the succeeding cycles; and that as soon as it accumulates to the amount of the $\frac{1}{m}$th of a vibration, which we have shown to be the distance of the most contiguous corresponding vibrations of the strings in the first cycle, it will bring these vibrations together, and cause a new coincidence for the first time. The number of cycles of the concord to produce this effect, neglecting the portion of the first cycle, is hence $= \dfrac{\frac{1}{m}}{n\left(\frac{s}{s'} \sim 1\right)}$, which, multiplied by the time of one of those cycles, or $\frac{m}{N}$, gives

$$\frac{1}{n\left(\frac{s}{s'} \sim 1\right)N}$$

for the interval, in seconds, between these distant coincidences, which is the *cycle of beats*, the beats being heard at the places of coincidence.

The time, in seconds, between two consecutive beats being therefore $\dfrac{1}{n\left(\frac{s}{s'} \sim 1\right)N}$, *the number of beats in one second*

$$= n\left(\frac{s}{s'} \sim 1\right)N \ldots\ldots (1),$$

which is independent of the numerator m of the fraction $\frac{m}{n}$ belonging to the concord.

The value of $\frac{s}{s'}$ depends upon the interval by which the concord is increased or diminished.

Let the number of *degrees* in the temperament, or the interval between the notes of s and s' be denoted by D° and, (art. 21,) we shall have

$$\mathrm{D}^\circ = \frac{\log s \sim \log s'}{\cdot 00041237};$$

consequently

$$\log \frac{s}{s'} \text{ or } \log \frac{s'}{s} = \cdot 00041237\ \mathrm{D}^\circ.$$

Now, since $\frac{s}{s'}$ is very nearly unity, we have, by the principles of logarithms,

$$\frac{s'}{s} \sim 1 = (\text{hyp. log} \frac{s}{s'} \text{ or hyp. log} \frac{s'}{s})$$

$$= (\log \frac{s}{s'} \text{ or } \log \frac{s'}{s})$$

$$\times\ (\text{hyp. log } 10 \text{ or } 2 \cdot 302585).$$

Therefore, by substitution,

$$\frac{s}{s'} \sim 1 = \cdot 0009495\ \mathrm{D}^\circ = \frac{\mathrm{D}^\circ}{1053 \cdot 2} \ldots\ldots (2),$$

which will be the same, whether the temperament be sharp or flat.

By substituting this value of $\frac{s}{s'} \sim 1$ in (1), we have for the calculation of the number of beats in one second, or ten seconds, the formula,

$$\text{Beats in 1 second} = \frac{n\,\mathrm{D}^\circ}{1053 \cdot 2} \cdot N = \frac{3\,n\,\mathrm{D}^\circ}{3160} \cdot N$$

$$\text{Beats in 10 seconds} = \frac{n\,\mathrm{D}^{\circ}}{105{\cdot}32} \,.\, N = \frac{3\,n\,\mathrm{D}^{\circ}}{316} \,.\, N$$

33. For actual computation it will be necessary to know also N, the number of vibrations per second of the lower string *k*. We shall here simply state the formula, in a form most convenient for calculation, which results from the mathematical theory of the vibrations of a stretched cord.

Let l = the length *in inches* of the cord or wire.
c = the length *also in inches*, whose weight would be equal to that by which it is stretched.

Then the number of vibrations per second will be

$$N = \frac{\sqrt{c}}{l} \times 9{\cdot}8257,$$

in which 9·8257 is $\frac{1}{2}\sqrt{g}$, g being the accelerative force of the gravity of the earth at its surface, expressed in inches.

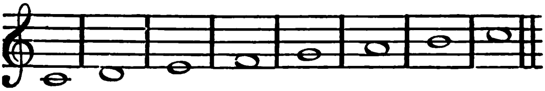
34. To determine thus the numbers of vibrations due to the several musical notes according to the usual concert pitch, I have recently made an experiment with a piece of the stoutest plain string of the pianoforte, which weighed very

nearly 2 grains to the inch; 272 inches weighed just 9 drams or 540 grains. A length of 7·28 inches, tended in a vertical position* by a weight of 28 avoirdupois pounds, sounded the common pitch-note A. In calculating the number of the vibrations of this note, according to the preceding formula, we hence find $c = 98726$ inches, $\surd c = 314{\cdot}2$ inches, $\surd c \times 9{\cdot}8257 =$ 3087·2 inches; and $N = \frac{3087{\cdot}2}{7{\cdot}28} = 424$. We, therefore, conclude that the pitch-note A vibrates about 424 times in one second. This may differ one or two vibrations from the truth, on account of the unavoidable small defects of the materials used in the experiment.

According to this determination, the vibrations, in one second, of each of the notes in the octave, ascending from C, will be:

---

* For such experiments it is very necessary that the string be in a vertical, and not in a horizontal, position. In the latter position, the string is prevented by friction from experiencing the full effect of the weight, and the calculation will always give the number of vibrations in excess of the true value. The above experiment was first made with the string tended horizontally over a very small pully and a well-defined bridge, and the result came out 470: the error was plainly attributable to the cause alluded to.



Vibrations} 254 286 318 339 382 424 477 509

and the vibrations in the next octave above will be double, and in the one below half, of these numbers.

35. The number of beats of an imperfect consonance in one second, according to the formula page 62, is $= N \div \frac{3160}{3\,n\,D^\circ}$. To find the divisor $\frac{3160}{3\,n\,D^\circ}$, the values of n for the concords (11,) are

Minor-third .... $n = 6$,
Major-third .... $n = 5$,
Fourth.................. $n = 4$,
Fifth .... $n = 3$,
Minor-sixth .... $n = 8$,
Major-sixth........... $n = 5$.

With these and the known temperaments of the systems noticed in the preceding pages, the values of this divisor are easily calculated;

and the results come out as in the following table:

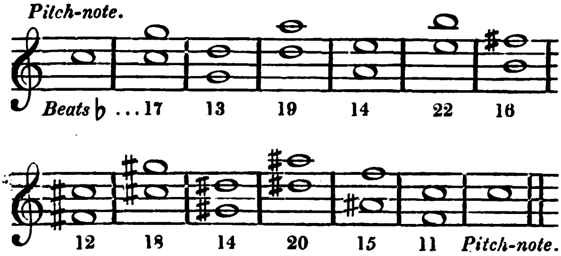
DIVISORS FOR BEATS IN ONE SECOND.

Concord.	12 equal Intervals.	19 equal Intervals.	31 equal Intervals.	50 equal Intervals.	53 equal Intervals.
Minor-third...	18·5 ♭	1948·5 ♯	48·4 ♭	79·3 ♭	215·4 ♯
Major-third...	25·3 ♯	47·0 ♭	442·1 ♯	149·7 ♭	245·9 ♭
Fourth	221·4 ♯	60·0 ♯	83·6 ♯	72·7 ♯	6345·4 ♯
Fifth	295·2 ♭	80·0 ♭	111·4 ♭	96·9 ♭	8460·5 ♭
Minor-sixth...	15·8 ♭	29·4 ♯	276·3 ♭	93·6 ♯	153·7 ♯
Major-sixth...	22·1 ♯	2338·1 ♭	58·1 ♯	95·1 ♯	258·5 ♭

With this table, we can readily find the number of beats of a concord in any part of the scale, by dividing the number of vibrations of the lower note by the corresponding divisor. Those which have ♯ attached, beat in consequence of being *sharper* than the true interval; and, on the contrary, those marked ♭, beat in consequence of the interval being *flatter* than perfect. If a perfect interval be tuned either sharp or flat by the same quantity, it will beat in about the same manner, and hence, in tuning by beats, it will always be necessary to first make the interval perfect, or nearly so, and then to bring it to the proper beats by elevating or depressing the upper note according as it is to be sharper or flatter than the true consonance. The nearer a concord is tuned to perfection, the slower it beats, until at last, when perfect, they entirely disappear, because $\frac{s}{s'} \sim 1$ approaches zero and vanishes.

As the fifth is the interval best adapted to the purposes of tuning, we shall here subjoin the beats of this concord, which ought to occur in tuning the scales of 12 and 19 equal intervals in the octave.

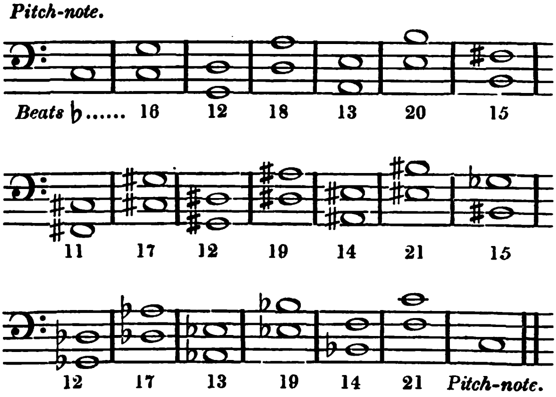
36. Numbers of beats in 10 seconds of the cycle of fifths, in tuning the chromatic scale of equal semitones.



This is the best system for the usual organs, &c. which divide the octave into 12 semitones.

It will be observed, that seven octaves are tuned downwards.

37. Numbers of beats in 10 seconds in tuning the cycle of fifths, according to the enharmonic scale of 19 equal intervals in the octave.


Pitch-note.
Beats ♭...... 16 12 18 13 20 15
11 17 12 19 14 21 15
12 17 13 19 14 21 Pitch-note.

C♭ } is the same as { B♯
F♭ } { E♯

This enharmonic scale of 19 sounds in the octave would furnish a very accurate diatonic series on each of them as a key-note; and the indices of these 19 keys would be *one flat* to *six flats*, *one sharp* to *twelve sharps*, and the natural key.

(NOTE.)

Sir John Herschel in his paper on Sound,* makes the following observation on the beats of imperfect concords: "Beats will likewise be heard when other concords, as fifths, are imperfectly adjusted. Suppose one string to make 201 vibrations, while the other makes 300, then, at and about the 100th of one, and the 150th of the other, the former will have gained half a vibration, and those vibrations of the one which fall exactly on those of the other, being performed with contrary motions, will destroy each other; those which fall intermediate, only partially."

The incautious reader would here be naturally led to suppose that, after the 100th vibration of the one, and 150th of the other string, this contrary motion of the successive vibrations occurs for the *first time.* This, however, is not the case; we have shown that the cycle of beats will be completed as soon as the two *most contiguous* vibrations in the cycle of the chord are brought to a coincidence. In the case of the fifth, these two most contiguous vibrations differ by only one-third of a vibration of the former string, and will become coincident after 67 vibrations of the former and 100 of the latter; this may be plainly seen by dividing the numbers 201 and 300 by 3. When 100 vibrations of one and 150 of the other are completed, a beat will, therefore, have interposed, and the vibrations will have passed through another half cycle of the beats, that is, the strings will have performed just a beat and a half. The contrary motion of the vibrations will *first* occur when the former string performs 33½, and the latter 50 vibrations.

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* Encyclopedia Metropolitana, vol. ii., mixed sciences, p. 794.

MISCELLANEOUS ADDITIONS.

38. According to the strict definition of sound (art. 1,) it can only exist in the ideas of the hearer; to avoid circumlocution, it is customary, however, to confound it with the vibrations that produce it. In the preceding pages we have, therefore, not hesitated in using the term either as denoting the vibrations themselves or the sensation they excite in the mind. We have also subjected all our calculations to the properties of the vibrations of musical strings, as most simple in their mathematical theory, though not the only agents capable of producing musical sounds. The production of sounds may be referred to two distinct kinds of agents, viz. the vibrations of elastic bodies, and the vibrations of columns of air, of definite extent, inclosed in cavities. Amongst the former, we may mention the vibrations of strings, such as those of violins, pianofortes, &c., the vibrations of tuning-forks, bells, reeds, musical-glasses, &c.; amongst the latter, may be noticed the vibrations of columns of air in tubes,

such as flutes, clarionets, trumpets, organ-pipes, &c.

39. If a tuning-fork* be made smartly to vibrate, and the two prongs presented to the eye, the ends will appear very sensibly thickened, but will gradually diminish again to their natural magnitude as the vibratory motion subsides. The vibrations may, however, be made more plainly visible by waving the fork, with moderate quickness, from side to side; a considerable number of images of the prongs will appear, arranged like the folds of a fan, which will expand more and more as the rapidity of the motion is increased.

This curious phenomenon, which extends also to other vibrations, seems to arise from this principle, that the eye, with the axis or eyeball fixed, can only receive the impression of an image, when an object capable of exciting that impression is presented for a definite time at a stated position. As an illustration of this principle, draw a fine black line, about one-fourth of an inch in length, on the middle of a sheet of white paper. Then, if the paper be waved backward and forward, so as to make the line move lengthways, it will, through all the intermediate positions, produce the impression of a continued line across the paper, because each point passed over is occupied by the line for a definite time, viz. the time the line takes in passing through its length, and this time is sufficient to produce the impression of blackness on the paper. But, if the paper be waved so as to make the line move sideways, no intermediate impression whatever will be produced, as no place is then occupied by the line for a sufficient time to produce an impression on the visual organs, except at the extreme positions where the line is for an instant stationary.

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* A tuning-fork being held to the ear when vibrating and turned gradually round, a sensible increase and diminution of the sound will be experienced; in a particular position, where the prongs are presented at an angle of about 45°, the sound is scarcely, if at all, perceptible.

In the case of the vibrating fork, we observe, that, when the waving and vibratory motions conspire in the same direction, the velocity is too rapid to produce any impression on the eye, and the quick return of the prong to its former position is prevented by the progressive motion; that, on the contrary, at particular points where the vibratory and progressive motions become equal and in contrary directions, the prong is for a moment stationary, and a distinct image is produced of that edge which follows in the direction of the progressive motion.

40. Every interval may be determined by combinations of the major-third, fifth, and octave. Thus, for

a comma,	ascend 4 fifths, descend 1 major-third and 2 octaves;
a minor* chromatic semitone,	ascend 2 major-thirds, descend 1 fifth;
a major* chromatic semitone,	ascend 3 fifths and 1 major-third, descend 2 octaves;
a diatonic semitone,	ascend 1 octave, descend 1 major-third and 1 fifth;
a minor-tone,	ascend 1 major-third, and 1 octave, descend 2 fifths;
a major-tone,	ascend 2 fifths, descend 1 octave;

and so may all other intervals be formed.

41. A violin being tuned by three successive perfect fifths, G D, D A, A E, a perfect double octave to the low open G, formed in the natural position with the second finger on the first string, will not form a perfect minor-third with

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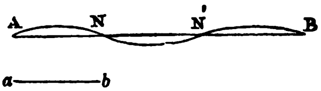
* A minor chromatic semitone occurs with the minor-tone, and a major chromatic semitone with the major-tone.

the E of the open string; the E will be too sharp by 13 degrees or a comma, as may readily be shown by the values on page 21. This departure from the diatonic series to the key of the primitive sound G, from which the fifths were tuned, which is by musicians commonly called the "break," is a formidable obstacle to successive perfect modulation into the key of the fifth ascending, on all instruments the sounds of which are fixed.

Violin players, who aim at perfect melody, should keep in mind that the same fingerings on the first and fourth strings do not constitute intervals of a perfect thirteenth; but that either the fingerings on the first string are a comma too sharp, or those on the fourth string a comma too flat. For instance, in the key of one sharp, founded on the open G string, it will be necessary in the performance of a solo to avoid the open E, and substitute the little finger on the second string. In double stopping, however, the open E string may be used to chord with the open A, but should be avoided in all others. Many similar considerations will naturally occur to the reader in relation to the execution of the different keys. It may be advised, as a general rule, to be very sparing in the use of the open strings.

42. In modulations, the following technical terms are sometimes used in reference to the original key, when any one of the notes assumes the character of a fundamental sound.

Key	Tonic.
2d	Supertonic.
3d	Mediant.
4th	Subdominant.
5th	Dominant.
6th	Submedient.
7th	Subsemitone.
Octave	Tonic.



43. If two strings A B, *a b*, of which the latter is just one-third of the former, be stretched with equal weights and placed parallel to each other, then, the string *ab* being put in a state of vibration, the vibrations will, by a peculiar sympathy, be transferred to the string A B, which will vibrate in three divisions, as in the figure, so as to conform exactly with the vibrations of the other string. That the string AB does so vibrate, may be satisfactorily shown by fastening a

small ring of thread loosely round it; if the ring be placed at either of the points N, N′, which divide the string into three equal parts, it will remain perfectly at rest, on the vibrations being communicated to *a b*; but, if it be removed to any other part of the string, it will be much tossed about by the excited motion of the string. It is of little consequence whether the string *a b* be one-third of A B; but it must be disposed to vibrate exactly three times as fast as A B, so that the sounds of both strings may form a perfect concord. If the string *a b* be two-thirds of A B, or if the string A B performs exactly 2 while *a b* performs 3 vibrations, in which case the fundamental sounds of the strings will form a perfect fifth, the same consequences will follow; *a b* being put in motion, A B will vibrate in 3, and *a b* will vibrate partially in 2 divisions.

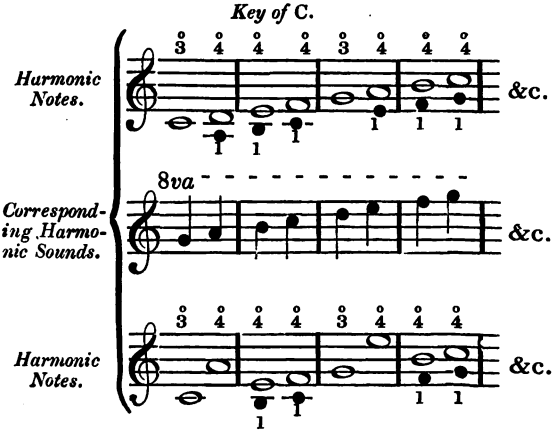
The experiment may be tried with any two strings whose vibrations bear a simple proportion to one another. If the vibrations be as 3 to 4, the former string will vibrate in 4, and the latter in 3 divisions.

This shows how very susceptible strings are to harmonical vibration. They seldom produce their fundamental sounds without being also agitated by some of these vibrations which produce harmonic sounds; they are also very susceptible to the communication of rotatory motions. The various combinations of these different modes of vibration must have a considerable influence on the musical quality and expression of the fundamental sound. The late Dr. Young, by throwing a beam of light upon the vibrating string of a violin, and marking the motion of the bright spot which it made, he found that the string very seldom vibrated in the same plane, but that the middle point described various complicated curves, according to different ways of drawing the bow.

As strings, which form concordant intervals, possess so powerful a sympathy in the reciprocation of their vibrations, it is very probable that those which form discordant and imperfect intervals must, on the contrary, have a reciprocally distorting influence on the modes in which they vibrate. It also appears, from this consideration, that imperfect concords are probably more disposed to be put out of tune than perfect ones; and a question naturally arises whether this jarring influence may not vitiate, in some degree, the general theory of beats.

The sympathy of the vibrations of strings extends to all other musical vibrations, and the phenomena generally come under the term, resonances of sound, or reciprocated vibrations of columns of air. As simple cases of other resonances may be noticed, the reciprocated vibrations of a sounding-board, when in contact with a vibrating string or tuning-fork; the resonance of an open pipe when a vibrating fork, having the same pitch note, is held over it; the resonance of a drinking glass or musical instrument to the sound of the voice, &c.

44. Example of the most simple harmonics on the violin.


Key of C.
Harmonic Notes.
o 3 o 4 o 4 o 4 o 3 o 4 o 4 o 4
1 1 1 1 1 1
&c.
8va
Corresponding Harmonic Sounds.
&c.
Harmonic Notes.
o 3 o 4 o 4 o 4 o 3 o 4 o 4 o 4
1 1 1 1
&c.

45. According to very careful experiments made by Fischer,* the numbers of vibrations in one second of the pitch-note A, or the open A on the violin, at the principal Berlin and Paris theatres, were as follow:

Berlin, Theatre	437·32	vibrations per second.
Paris, French Opera	431·34	
... Comic Opera	427·61	
... Italian Opera	424·17	

46. The pitch of a fork is made sharper by filing a little off at the ends of the prongs; and is made flatter by filing a little off the inner sides. The prongs should be made as nearly alike as possible, as any difference in their vibrations must impoverish the sound.

The pitch of the sound of a cylindrical pipe depends upon its length, according to the same principles as those of the vibrating string; it is, therefore, sharpened when made shorter, and flattened when made longer. The pitch of the sound of a conical pipe depends also on the proportion of the diameter of the base to its length; it is sharpened by diminishing either the diameter of the base or the length, and *vice versa;* the particular form of the pipe determines the quality or character of the sound produced.

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* Memoirs of the Berlin Academy for 1823, page 187.

The sound of an open metal pipe will be flattened or sharpened a little by bending a small part of the metal at the open end, a little inwards or outwards, respectively.

The sound of a stopped metal pipe will be flattened or sharpened a little by bending the ears, at the sides of the mouth, a little inwards or outwards, respectively.

The sound of an open wooden pipe will be flattened or sharpened a little by depressing or raising the leaden plate that hangs over the open end, respectively.

The sound of a stopped wooden pipe will be flattened or sharpened by drawing the plug outwards or forcing it inwards, respectively.

The sound of a reed-pipe will be flattened or sharpened by causing the brazen tongue to be lengthened or shortened, respectively.

47. Change the first string of a violoncello for another, the same as the second, that their sounds, having the same strength, may beat stronger and more distinctly. Then screw up this string, and, as it gradually approaches to a

unison with the second, the two sounds will be heard to beat, very rapidly at first, then gradually slower and slower, till at last they make a perfect unison without any beats whatever. When thus perfectly tuned in unison, if either string be put in vibration by the bow or finger it will excite vibrations also in the other, plainly, visible to the eye.

Alter very slightly the tension of either string, and their sounds will beat again. But now the motion of one string struck alone makes the other only start, and excites no regular vibrations, because the strings are not isochronous, that is, they do not vibrate exactly in equal times. And, the sounds being both drawn out with an even bow, not only an audible but a visible beating and irregularity is observable in the vibrations, which before were free and uniform.

Measure the length of one of the strings between the nut and the bridge, and, when they are tuned in perfect unison, at the distance of one-third of that length from the nut, mark that string with a speck of ink. Then placing the edge of your nail on the speck and pressing it to the finger-board, upon sounding the remaining two-thirds with the other string open, the sounds will form the concord of a perfect fifth, the ratio of which is $\frac{2}{3}$. But, upon moving the finger-nail a little upwards or downwards, the interval will become tempered a little sharp or flat, and, in both cases, the imperfect fifth will beat quicker or slower, as the interval is more or less altered.

The sounds of an organ being generally more uniform than any other, their beats are accordingly more distinct; and they are perfectly regular when the blast of the bellows is uniformly sustained.

Beats are also pretty audible on the harpsichord, when every thing else is silent, especially while the sounds are vanishing. They are remarkably disagreeable in a concert of strong treble voices, when some of them are out of tune; or in a ring of bells ill tuned; or in a full organ badly tuned, particularly on the compound stops, called the Cornet and Sesquialter, and all other shrill and loud stops when mixed with the rest.

48. The vibrations of two sounds, which form

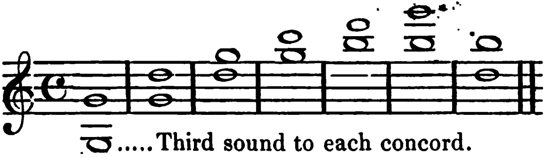
a concord, coincide at short and regular periods. These coincidences, like a new series of vibrations, have the effect of producing on the ear a slight impression of a new sound, which is called the "third-sound." This third-sound, though very faint, may be distinctly heard with the simpler concords when correctly tuned; and the time of its vibration is evidently the same as the cycle of the concord. (See p. 59.) Let the interval of the concord be determined by the fraction $\frac{m}{n}$; then, in the cycle of the concord,

The third-sound	performs	1 vibration.
The lower sound of the concord		m vibrations.
The upper sound of the concord		n vibrations.

The sounds of the concord are, therefore, harmonics to the third-sound, according to numbers of divisions respectively equal to m and n. By hence referring with the values of m (page 9,) to the list of harmonic sounds (page 30,) we readily find the third-sound to each of the concords, in relation to the lower note, as a fundamental, to be as follow:

Concord.	Third-Sound.	
Minor-third......	2 octaves and a major-third,	descending.
Major-third......	2 octaves,	
Fourth	1 octave and a fifth,	
Fifth...............	1 octave,	
Minor-sixth......	2 octaves and a major-third,	
Major-sixth......	1 octave and a fifth,	

EXAMPLES.



.....Third sound to each concord.

All these concords are accompanied by the same third-sound, low G.

NOTE to Articles 36 and 37.

IN tuning the musical scale by the beats of imperfect fifths, it may be observed that, at those places where a perfect octave is tuned downwards, the number of beats of the fifth ascending must correspond with that of the fourth descending. By attending to this rule the octave may in some cases be adjusted more accurately than could otherwise be effected with the assistance of the ear only.

FINIS.

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